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Inhomogeneous model envelopes with helium enriched outer layer for BQ Ser, a double-mode Cepheid, are constructed with T_{eff}=6450K estimated from the observed spectral type. Periods of those models are compared with the observed period and period ratio. The mass of model satisfying its periods and the resonance condition is less than that derived from the evolution theory.

Keywords: Double-mode Cepheids, BQ Ser, Stellar pulsation.

§1. Introduction

Double-mode Cepheids are of great important since the pulsation theory has not succeeded in explaining their period ratio yet. The masses of these stars derived from the period ratio are smaller than expected from the standard evolutionary calculations. These mass anomalies for double-mode Cepheids pointed out by Rodgers,[1)] and stressed by Petersen[2)] and Takeuti.[3)] One of the attempts to explain this mass discrepancy is to change the chemical composition of outer envelopes. The two zone envelope models are introduced by Cox, Deupree, King, and Hodson[4)] and Cox, Michaud, and Hodson[5)] and extensively studied by Cox et al.,[6)] Petersen,[7,8)] for the double mode Cepheids. The ratio of two observed periods for the double-mode Cepheids is well known to lie in the narrow range 0.697-0.711 as summarized by Stobie.[9)] Simon[10)] has proposed that the stable double-mode pulsation is caused from resonance effects among the fundamental mode, the first overtone, and the third overtone.

Concerning with Cogan's criticism[11)] on the double-layer model proposed by Cox and his collaborators, Takeuti and Aikawa[12)] have studied double-layer theory much more carefully and pointed out that the resonance condition is satisfied by the models having the mass less than that derived from the standard evolutionary calculations. Their study has been very interesting unless parameters chosen by them are rather restricted. In the present paper the two zone model envelopes are constructed for BQ Ser, one of few observed double-mode Cepheids in the northern sky, with the observed effective temperature and determine the mass and luminosity consistent with its period and period ratio to check whether or not the results of Takeuti and Aikawa[12)] are adequate to the real star. Its observed period here used is adopted from Szabados.[13)] The spectral type for estimating T_{eff} is adopted from Uji-iye.[14)] The observational data used in this paper summarized in Table 1.

Table 1. Summary of the adopted observational data for BQ Ser

P_F (day)	4.27073	log P_F	0.6305
P_{1H} (day)	3.012	P_{1H}/P_F	0.7052
Spectral type	F6III	T_{eff}	6450K

§2. Models and Periods

Our envelope models constructed for BQ Ser are two-zone models with helium enriched outer layer, with the depth of mixed zone and helium abundance as parameters. The outer zone is limited at the layer whose boundary temperature is T_B. In present calculations we adopted T_B=63000K as a shallow mixed zone allowing for Petersen.[7] Taking into consideration of Cox et al.,[6] we also calculated with T_B=250000K as a deep one. In order to estimate the chemical composition in the surface layer, we make allowance for the result of Takeuti and Aikawa.[12] Here expressing the hydrogen, helium and heavy elements abundance in the outer layer by X_S, Y_S and Z_S respectively, we suppose three cases of Y_S=0.4, 0.5, 0.7. The initial abundance bellow the bottom of outer layer, (X_O, Y_O, Z_O), is (0.70, 0.28, 0.02) and we assume $Z_S/X_S=Z_O/X_O$. The Stothers-Simon formula[15] is used to calculate opacities. The ratio of mixing length to the pressure scale height is assumed to be unity and T_{eff} of 6450K is adopted for all models.

We obtained the period of fundamental mode P_F, the period of first overtone P_{1H}, and the period of third overtone P_{3H} for BQ Ser from the linear adiabatic pulsation theory. Results on the period and period-ratio diagram are shown in Figure 1. The normalized resonance distance $d(0+1,3)=1 - P_{3H}/P_F - P_{3H}/P_{1H}$ defined by Petersen[7] which is identical with d_3 in Simon,[10] are shown in Figure 2.

§3. Results and Discussion

The masses and luminosities of BQ Ser are derived from our models by comparison with the observed period and period ratio. These results are given in Table 2 together with the surface gravity calculated from them. We could not find the mass satisfying observed periods in our given mass range of the inhomogeneous deep model of Y_S=0.4 and 0.7. In the former case somewhat large mass compared with the evolutionary one seems obtainable by fitting, but resonance distance d(0+1,3) is not adequate for sufficient resonance. In the later case the resonance condition is satisfactory, but fitting mass looks as small as the homogeneous model. The mass obtained for the model of Y_S=0.5 is nearly equal to the evolutionary one both shallow and deep mixed zone. These mass and luminosity fulfil Becker, Iben and Tuggle's[16] M-L relationship for (Y,Z)=(0.28,0.02). The resonance distance in these cases deviates from the resonant center d(0+1,3)=0 however, then strong resonance is unfavorable. In

Table 2. Mass, luminosity and surface gravity of BQ Ser for various models fitting the observational data

Model		$M/M_{\odot}$	$\log L/L_{\odot}$	$\log g$
Homogeneous model	Y_S=0.28	1.86	3.03	1.87
Inhomogeneous shallow model (T_B=63000K)	Y_S=0.4	3.63	3.26	1.93
	Y_S=0.5	5.21	3.37	1.98
	Y_S=0.7	6.40	3.43	2.01
Inhomogeneous deep model (T_B=250000K)	Y_S=0.5	5.33	3.39	1.96

the present models given in Table 2, the homogeneous one and inhomogeneous shallow one of Y_S=0.4 barely fulfil the resonance condition $|d(0+1,3)|<0.05$. Taking shallower mixed zone, we may obtain the adequate model whose resonance condition become stronger than present one.

In the present paper, we have checked the double-layer model with the resonance hypothesis using the observed properties of BQ Ser. In conclusion we may note that the model of Y_S=0.4 with the mass less than that derived from the standared evolutionary calculations is favorable to satisfy the pulsation properties and the resonance hypothesis. This is in agreement with the result of Takeuti and Aikawa.[12)] If double-mode Cepheids do not present any new type distinct from the single-mode classical Cepheids, the disagreement of masses appeared again in the present study is likely the evidence for the defect of recent stellar pulsation theory.

The surface gravity for BQ Ser is less than 2.0 dex by the analysis performed in the present study. This value is not so different from that of another beat Cepheid U TrA obtained by Rodgers and Gingold.[17)] They match with the luminosity classes of single-mode Cepheids which are I-II. On the contrary, larger surface gravity was obtained for a beat Cepheid TU Cas by Schmidt,[18)] which seems favorable to the helium enriched envelope as discussed by Takeuti.[19)] If the surface gravity is larger than 2 dex in observations for BQ Ser, it is likely that the difference indicates the apparent increase of surface gravity caused from helium enrichment in the atmosphere. It is very interesting whether the luminosity classes of beat Cepheids are the same as those of single-mode ones or not. Although Herbig[20)] has once reported that the luminosity calss of BQ Ser is III, further spectroscopic data are expected to confirm it.

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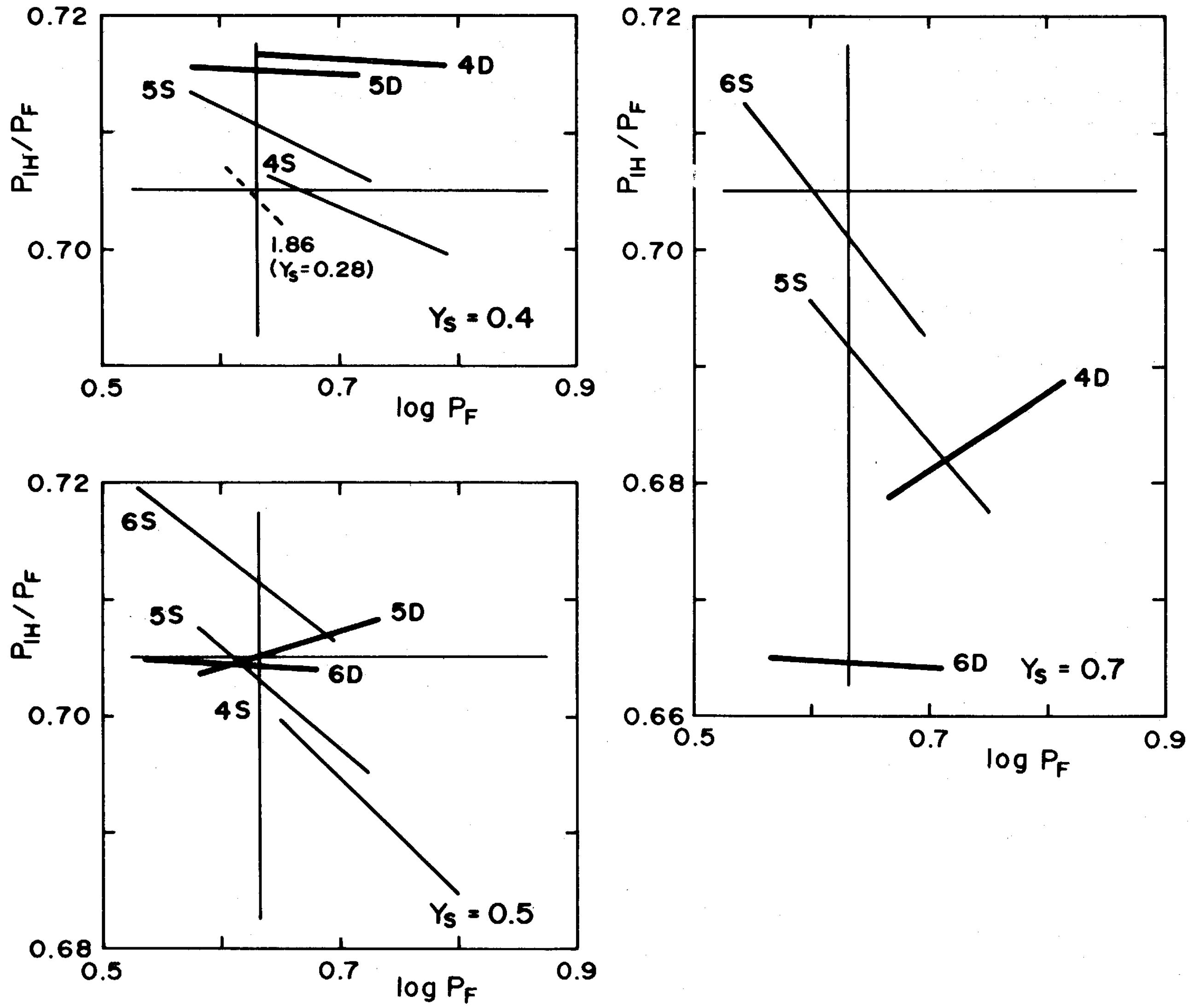

Fig. 1. The period and period ratio diagram. Vertical and horizontal lines mark the observed period and period ratio of BQ Ser, respectively. Narrow and thick lines show those with the shallow(S) and deep(D) inhomogeneous envelope models, respectively. Numerical values indicate masses of models. Right and left edges correspond to models with $L=2000L_\odot$ and $3000L_\odot$, except the homogeneous model(dotted line).

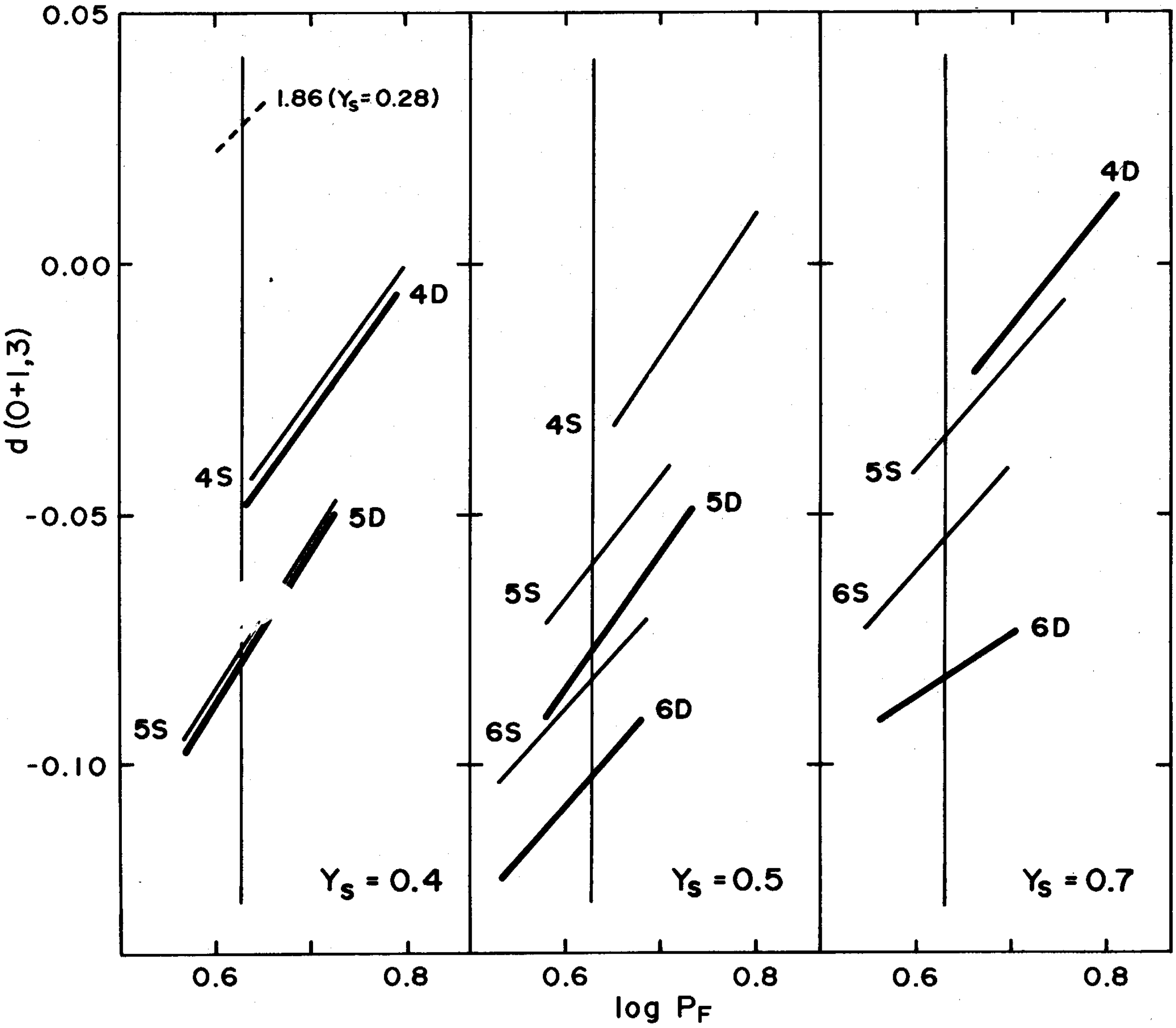

Fig. 2. The normalized reasonance distance. Expressions are the same as Figure 1.

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